Calculation of Steady and Unsteady Pressures on Wings at Supersonic Speeds with a Transonic Small-Disturbance Code

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A transonic unsteady aerodynamic and aeroelasticity code has been developed for application to realistic aircraft configurations. The code is called CAP-TSD (computational aeroelasticity program—transonic small-disturbance) and uses a time-accurate approximate factorization algorithm for solution of the unsteady transonic small-disturbance equation that is efficient for solution of steady and unsteady transonic flow problems including supersonic freestream flows. This code can treat complete aircraft geometries with multiple lifting surfaces and bodies. Applications to wings in supersonic freestream flow are presented. Comparisons with selected exact solutions from linear theory are presented, showing generally favorable results. Calculations for both steady and oscillatory cases for the F-5 wing and the Royal Aerospace Establishment tailplane models are compared with experimental data and also show good overall agreement. Selected steady calculations are also compared with the results from a steady flow Euler code.

Nomenclature

c = airfoil chord
$c_{l\alpha}$ = section lift-curve slope
c_r = wing reference chord
C_p = pressure coefficient
$\tilde{C}_p$ = unsteady pressure coefficient normalized by oscillation amplitude
f = function defining position of lifting surface
k = reduced frequency, $= \omega c_r/2U$
M = freestream Mach number
t = time, nondimensionalized by U/c_r
U = freestream velocity
α = angle of attack
γ = ratio of specific heats
Δt = nondimensional time step
$\bar{\eta}$ = fractional semispan coordinate
θ = phase angle between unsteady lift and pitch angle
ϕ = disturbance velocity potential
ω = angular frequency

Subscripts
0 = mean value
1 = dynamic value

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Introduction

CURRENTLY, there is considerable interest in the development of methods for calculating supersonic unsteady aerodynamics for application to advanced configurations. Although flutter is oftentimes critical at transonic speeds, critical conditions can also be encountered at both low and high supersonic speeds, depending on the configuration and its operating envelope. Linear theory has been the primary analytical too for analyzing flutter in the low supersonic region, with recent efforts directed toward refining the potential gradient methods (Refs. 1 and 2, for example). In linear theory, wing loading is determined by the relationship of the Mach lines to the planform. The Mach lines, which are infinitesimal approximations to shock waves, tend to lose significance for planforms with bluntness, thickness, or angle of attack as finite-strength shock waves must be treated. In this study, a different approach is used to calculate unsteady aerodynamics for aeroelastic analysis. A finite-difference technique is used to solve the transonic small-disturbance flow equation by making use of shock capturing to treat wave discontinuities. Thus, the nonlinear effects of thickness and angle of attack are included.

Such an approach is made feasible by the development of an efficient code by the Unsteady Aerodynamics Branch of the NASA Langley Research Center. The code is called CAP-TSD (computational aeroelasticity program—transonic small disturbance) and is based on a fully implicit approximate factorization (AF) finite-difference method to solve the time-dependent transonic small-disturbance equation.[3,4] The time-accurate AF algorithm is very stable and is efficient on current supercomputers with vector arithmetic. A similar TSD code, called XTRAN3S,[5] has also been developed that used a different numerical algorithm and has been applied primarily to subsonic freestream applications. Supersonic applications of a two-dimensional code for the low-frequency TSD equation have also been made.[6]

The emphasis in the development of CAP-TSD code has been on the capability of analyzing complete configurations in the low transonic range. Calculations for several configurations, including a complete F-16C model, have been made and are presented in Ref. 4. Only a brief overview of the analysis and computer program pertinent to the results presented herein is given, because the analysis have been described elsewhere.[3,4]

This paper presents the application of the CAP-TSD code to the calculation of steady and unsteady flows for low-to-moderate supersonic Mach numbers. In particular, comparisons with exact linear theory solutions are made for steady and unsteady cases to evaluate the shock capturing and other features of the current method. In addition, steady solutions obtained from an Euler code[7] are used to evaluate the small-disturbance aspects of the code. Steady and unsteady pressure comparisons are made with measurements for an F-5 wing model[8] and for the Royal Aerospace Establishment (RAE) tailplane model.[9]

CAP-TSD Program

The CAP-TSD program is a finite-difference program that solves the general-frequency modified TSD equation. The TSD potential equation is defined by

$$M^2 (\phi_t + 2\phi_x)_t = [(1-M^2)\phi_x + F\phi_x^2 + G\phi_y^2]_x + (\phi_y + H\phi_x\phi_y)_y + (\phi_z)_z \quad (1)$$

Several choices are available for the coefficients F, G, and H, depending upon the assumptions used in deriving the TSD equation. For the applications herein, the coefficients are defined as

$$F = \tfrac{1}{2}(\gamma + 1)M^2 \quad (2a)$$

$$G = \tfrac{1}{2}(\gamma - 3)M^2 \quad (2b)$$

$$H = -(\gamma - 1)M^2 \quad (2c)$$

The linear potential equation is recovered simply by setting F, G, and H equal to zero.

Equation (1) is solved within CAP-TSD by a time-accurate AF algorithm developed by Batina.[3] In Refs. 3 and 4, the AF algorithm was shown to be efficient for application to steady or unsteady transonic flow problems. It can provide accurate solutions in only several hundred time steps, yielding a significant computational cost savings when compared with alternative methods. Recently, several algorithm modifications have been made that improve the stability of the AF algorithm and the accuracy of the results.[10-13] These algorithm modifications include: 1) Engquist-Osher (E-O) type-dependent differencing to treat more accurately and efficiently the regions of supersonic flow; 2) extension of the E-O switch for second-order-accurate upwind differencing in supersonic regions to improve the accuracy of the results; 3) nonreflecting far-field boundary conditions for more accurate unsteady applications; 4) several modifications that accelerate convergence to steady state; 5) entropy and vorticity modifications for treating strong shock waves more accurately; and 6) supersonic outer boundary conditions. The capabilities available and used in this study included only the E-O switch. The CAP-TSD program can treat configurations with combinations of lifting surfaces and bodies, including canard, wing, rectangular vertical tails, control surfaces, tip launchers, pylons, fuselage, stores, and nacelles. Steady and unsteady pressures have been presented for several complex aircraft configurations in Ref. 4. The calculated results were in good agreement with available experimental pressure data, which validated CAP-TSD for multiple-component applications with mutual aerodynamic interference effects.

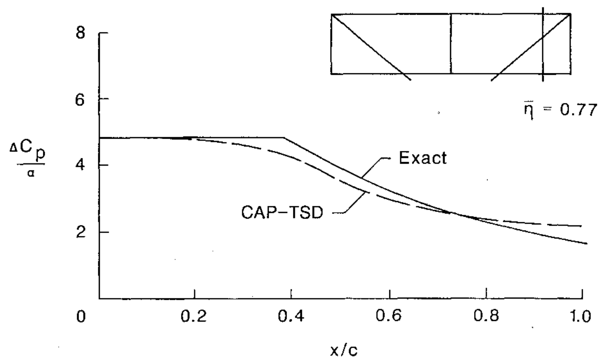

Fig. 1 Comparison between CAP-TSD and exact linear theory load distribution for rectangular wing of aspect ratio 4 at $M = 1.30$.

Results and Discussion

Several test cases were run to demonstrate the capability of CAP-TSD to treat flows with a supersonic freestream. First, comparisons with exact analytical solutions from linear theory are presented, followed by comparisons with experimental data for two wings.

For all of the cases presented here, a finite-difference grid was used that consisted of $90 \times 30 \times 60$ points in the x-y-z directions, giving a total of 162,000 grid points. The grid extended 10 root chord lengths ahead and aft of the wing, nearly 13 chord lengths above and below the wing, and one semispan outboard of the wingtip. On the wing, 50 equispaced points were used along each chord, and along the span 20 points were used clustered near the tip. This grid is one that would be used for a subsonic freestream and extends further than may be necessary for the supersonic cases. For the cases considered herein, the "reflecting" outer boundary conditions were used, and care was taken to ensure that the wing is located within the "Mach diamond" such that waves were not reflected from the outer boundaries back onto the wing surface. Running such a large grid is somewhat inefficient, but not having to readjust the grid at each Mach number is highly desirable. Calculations for supersonic Mach numbers lower than those considered herein require use of the appropriate characteristic boundary conditions for supersonic freestream flow, which were developed subsequently for CAP-TSD.[13] This grid might be considered a medium-to-coarse grid, and some improvements in the results might be noted if additional grid refinements were made.

For the oscillating wings, the calculations were made using 360 steps per cycle, which corresponded to a nondimensional time step Δt on the order of 0.1. Only two cycles of motion were calculated because the flowfield converges rapidly for supersonic flow. The second cycle was used for Fourier analysis. One subiteration per step was used for flowfield convergence.

For the steady flow calculation, time-step cycling was used, and the convergence was achieved in a few hundred time steps, normally about 250.

Comparisons with Linear Theory

Tip Loading in Steady Flow

The loading within the tip Mach line for a wing with a supersonic leading edge has an exact conical flow solution.[14] A typical result is shown in Fig. 1 and compared with a corresponding CAP-TSD calculation for a rectangular wing. For this case, the loading has a discontinuous slope at the Mach line that is smeared by the finite-difference scheme, but the overall trend of the loading is reproduced.

For a wing with a subsonic leading edge, the loading at the tip Mach line has a jump.[14] A loading of this type is shown in Fig. 2 for a hexagonal wing of 40-deg leading-edge sweep, taper ratio of 0.50, and $M = 1.12$. The overall trend of the loading again is given by CAP-TSD, but there is considerable smearing of the jump discontinuity. For this severe test case, further development of the finite differencing is desirable, but the agreement of the overall trend is encouraging.

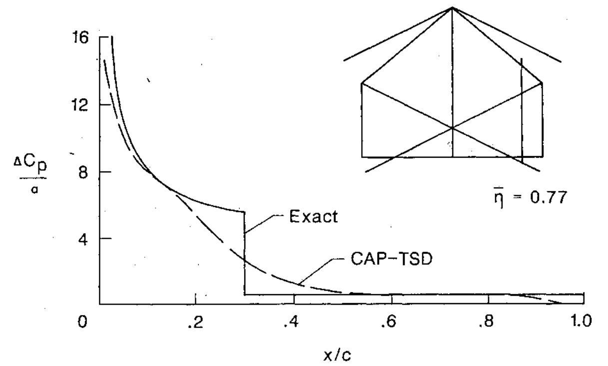

Fig. 2 **Comparison between CAP-TSD and exact linear theory load distribution for swept wing with 40-deg sweep and taper ratio of 0.5 at $M = 1.12$.**

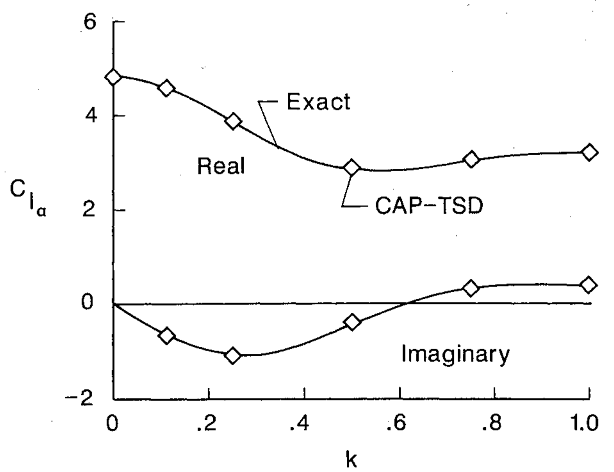

Fig. 3 Comparison between CAP-TSD and exact linear theory, unsteady lift-curve slope for two-dimensional flat plate at $M = 1.30$ oscillating in pitch about 41.3% chord.

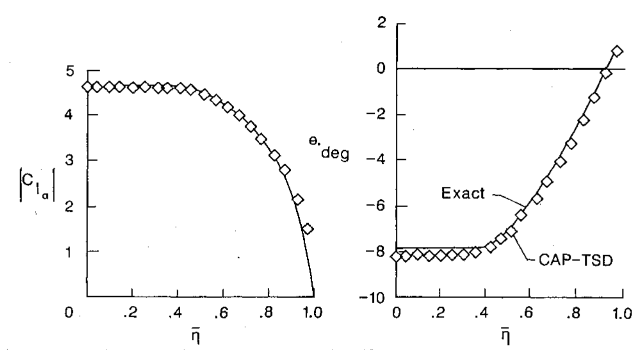

Fig. 4 Comparison between CAP-TSD and linear theory for the spanwise unsteady life magnitude and phase for an aspect-ratio-4 rectangular wing at $M = 0.111$.

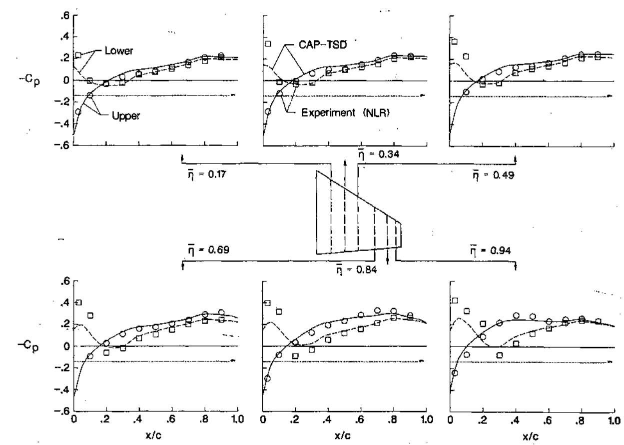

Fig. 5 Comparison between CAP-TSD and measured steady pressures for the F-5 wing model at $M = 1.10$ and $\alpha_0 = 0$ deg.

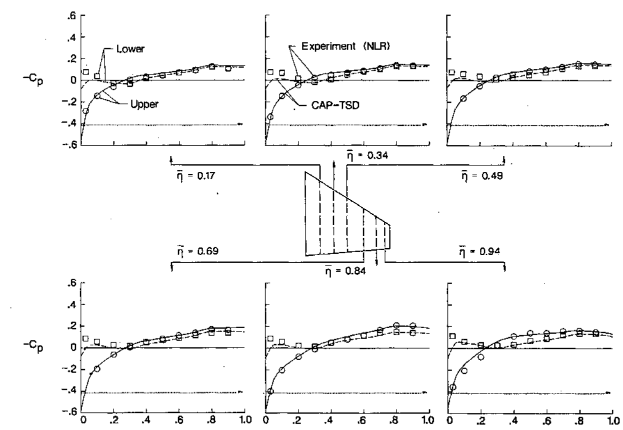

Fig. 6 Comparison between CAP-TSD and measured steady pressures for the F-5 wing model at $M = 1.32$ and $\alpha_0 = 0$ deg.

Unsteady Flows for Flat-Plate Wings

Exact solutions for a two-dimensional flat-plate airfoil are given in Ref. 15. Solutions for a rigid rectangular wing with an aspect ratio of 4 oscillating in pitch at $M = 1.30$ about 41.3% chord are presented in Ref. 16. Several cases have been run with CAP-TSD for this rectangular wing. Since the inboard solutions can be compared with the two-dimensional results of Ref. 15. The comparison for unsteady lift is shown in Fig. 3. The agreement with the exact solution up to $k = 1.0$ is excellent.

The spanwise distribution of lift magnitude and phase over the oscillating with the $k = 0.111$ is shown in Fig. 4. Good agreement is evident, with some overprediction at the tip. Note that the phase angle is shown with a highly expanded scale. These cases indicate that the overall loading is well reproduced by CAP-TSD in supersonic unsteady flows.

Results for the F-5 Wing

The F-5 wing model is a typical supersonic wing with a panel aspect ratio of 1.58, a leading-edge sweep angle of 31.9 deg, and a taper ratio of 0.28. The airfoil section is a modified NACA 65A004.8, which has a slightly drooped nose and is symmetric aft of 40% chord. Subsonic and transonic calculations for this model have been made using TSD codes[3,4,11,17,18] and generally have been in good agreement with the experimental data. Calculations with CAP-TSD are compared with steady flow data[8] for $\alpha_0 = 0$ deg and for Mach numbers of 1.10 and 1.32 in Figs. 5 and 6, respectively. Good agreement is demonstrated at both Mach numbers. For $M = 1.10$, there is a shock on the lower surface near the leading edge that is swept aft slightly more than the leading edge. Some deviation of the results obtained with CAP-TSD from the data in this region is noted. A further comparison for $M = 1.10$ with the results from a steady flow Euler code (FLO57MG) is given in Fig. 7. The Euler calculation uses a C-type grid that wraps around the nose of the airfoil and is able to resolve the leading-edge shock in more detail. The pressures over the remainder of the wing are in reasonable agreement, taking into account the coarseness of the grid for the Euler calculation (approximately 24,000 points).

Unsteady comparisons for one reduced frequency at each of the Mach numbers are given in Figs. 8 and 9 for pitching oscillations about $0.50c_r$. For $M = 1.32$, there is good agreement, and a typical supersonic loading is evident with only small variations along the chord. For $M = 1.10$, similar supersonic loading exists on the upper surface. In the region of the lower surface shock, however, a large peak in the unsteady loading is apparent, which appears to be due to an embedded transonic flow. CAP-TSD gives reasonable trends for this case, even for the very large unsteady loading at the tip station.

Results for RAE Tailplane

The RAE tailplane[9] is a planform that is typical of a tailplane for a supersonic fighter. It has a panel aspect ratio of 1.20, a taper ratio of 0.27, and a leading-edge sweep angle of 50.2 deg. The airfoil is approximately a NACA 64A010.2, which is thicker (10.2%) than is typical for supersonic wings. Only upper surface pressures were measured for this model.[9] Subsonic and transonic calculations for this model using XTRAN3S and a coarse grid were presented in Ref. 19.

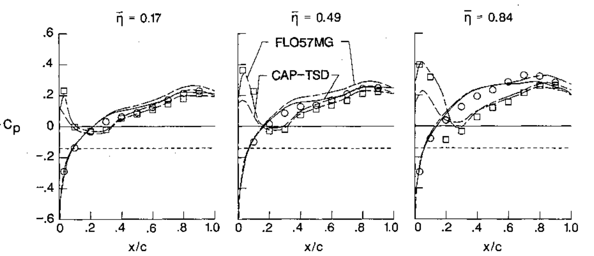

Fig. 7 Comparison among CAP-TSD, FLO57MG, and measured unsteady pressures for the F-5 wing model at $M = 1.10$ and $\alpha_0 = 0$ deg.

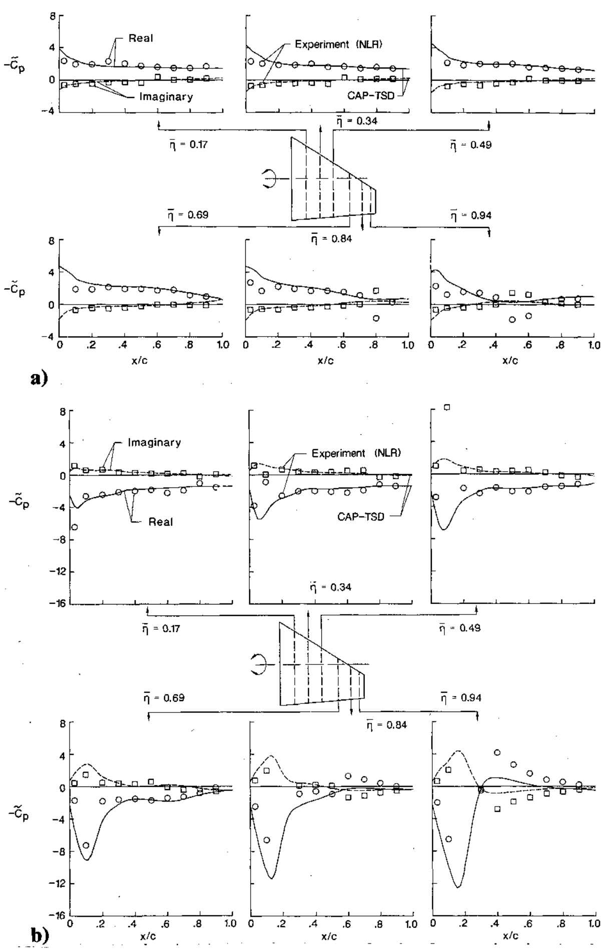

Fig. 8 Comparison between CAP-TSD and measured unsteady pressures for the F-5 wing model oscillating in pitch at $M = 1.10$, $\alpha_0 = 0$ deg, and $\alpha_1 = 0.267$ deg: a) upper surface; b) lower surface.

Calculated steady results from CAP-TSD for $\sigma_0 \approx 0$ and $M = 1.20$ and 1.71 are compared with measured pressures in Figs. 10 and 11. The measured pressures shown are the mean values measured at 3 Hz ($k < 0.015$), since steady pressures were not measured for these Mach numbers. The agreement is good, with some deviation shown near the tip at $M = 1.71$. Corresponding calculations for $M = 1.71$ with FLO57MG using the Euler equations are presented in Fig. 12 and show similar results, including the deviation near the tip.

Unsteady pressures for pitching oscillations about $0.682c_r$ at 70 Hz are presented for $M = 1.20$ and 1.71 in Figs. 13 and 14, respectively. The unsteady loading at $M = 1.20$ has more of a peak near the leading edge than the case at $M = 1.71$. The CAP-TSD results again are in good agreement with the measured data.

Some measurements were made at 5-deg mean angle of attack for this model. The steady pressures for $\alpha_0 = 5$ deg and $M = 1.71$ are compared with results from CAP-TSD in Fig. 15. Both the thickness loading and the pressure difference are well predicted even at this large angle of attack and Mach number.

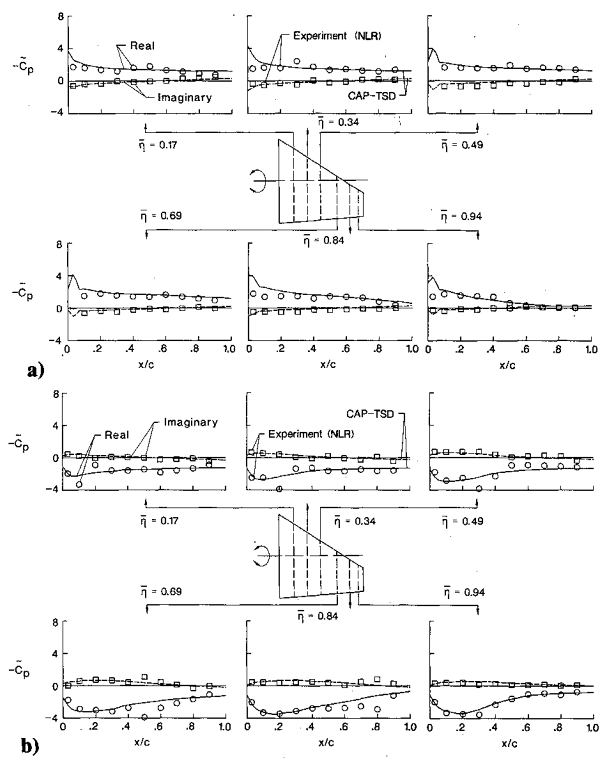

Fig. 9 Comparison between CAP-TSD and measured unsteady pressures for the F-5 wing model oscillating in pitch at $M = 1.32$, $k = 0.198$, $\alpha_1 = 0$ deg, and $\alpha_1 = 0.222$ deg: a) upper surface; b) lower surface.

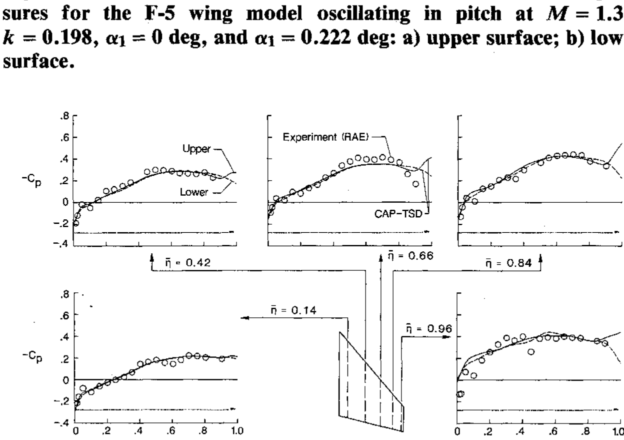

Fig. 10 Comparison between CAP-TSD and measured unsteady pressures for the RAE tailplane model at $M = 1.20$ and $\alpha_0 = 0.0$ deg.

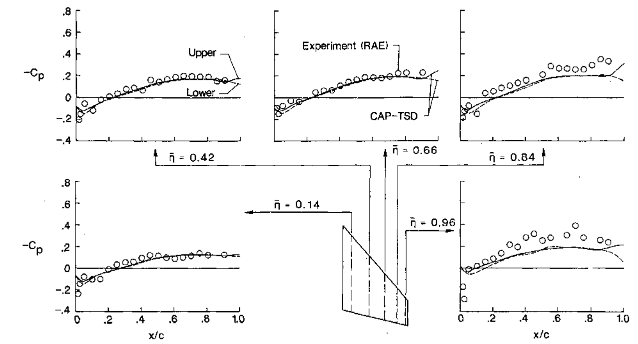

Fig. 11 Comparison between CAP-TSD and measured steady pressures for the RAE tailplane model at $M = 1.71$ and $\alpha_0 = 0.14$ deg.

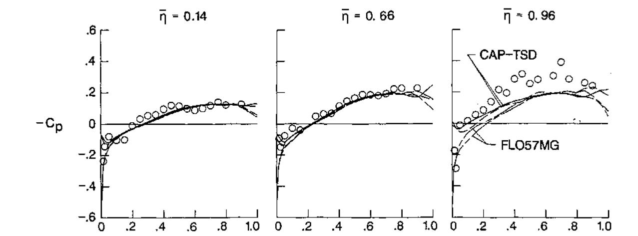

Fig. 12 Comparison between CAP-TSD, FLO57MG, and measured steady pressures for the RAE tailplane model at $M = 1.17$ and $\alpha_0 = 0.14$ deg.

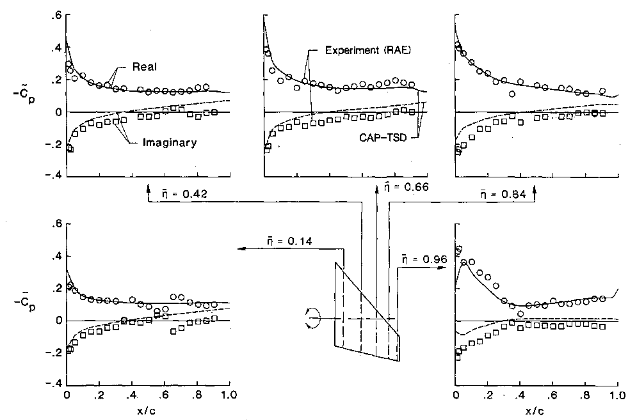

Fig. 13 Comparison between CAP-TSD and measured unsteady pressures for the RAE tailplane model at $M = 1.20$, $\alpha_0 = 0.0$ deg, $\alpha_1 = 0.378$, and $k = 0.346$ (70 Hz).

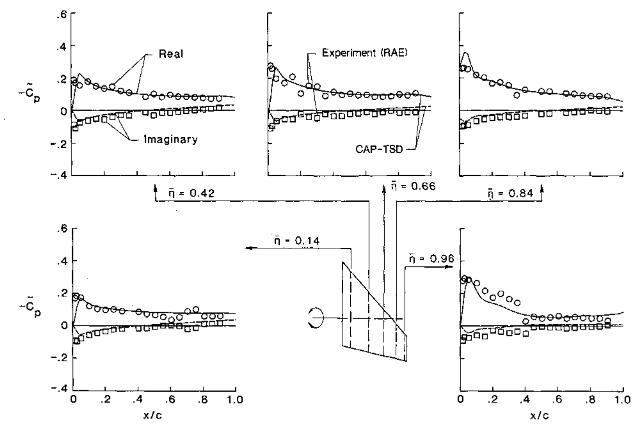

Fig. 14 Comparison between CAP-TSD and measured unsteady pressures for the RAE tailplane model at $M = 1.17$, $\alpha_0 = 0.014$ deg, $\alpha_1 = 0.570$ deg, and $k = 0.270$ (70 Hz).

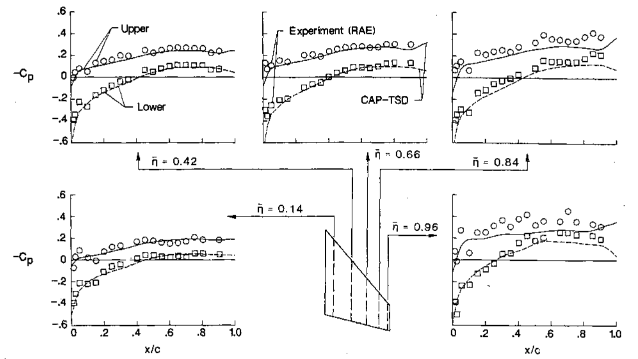

Fig. 15 Comparison between CAP-TSD and measured steady pressures for the RAE tailplane model at $M = 1.71$ and $\alpha_0 = 5.14$ deg.

Overall, the calculations with CAP-TSD are in good agreement with the experimental data for the thick RAE tailplane model. The correlation for these supersonic cases is comparable to or better than those obtained in the lower subsonic-transonic range with XTRAN3S using a coarse grid.[19]

Concluding Remarks

A transonic unsteady aerodynamic and aeroelastic analysis code called CAP-TSD (computational aeroelasticity program—transonic small disturbance) has been developed for application to realistic aircraft configurations. The code permits the calculation of unsteady flows about complete aircraft configurations for aeroelastic analysis in the flutter-critical transonic-speed range. The CAP-TSD code uses a time-accurate approximate factorization (AF) algorithm for solution of the unsteady transonic small-disturbance equation. The AF algorithm has been shown to be very efficient for steady or unsteady transonic flow problems, including supersonic freestream flows.

Results were presented for several wings that demonstrate the applicability of CAP-TSD for supersonic flows, including embedded transonic flows. Comparisons with known analytical solutions from linear theory demonstrated that CAP-TSD gives reasonable good fidelity to approximating weak shock waves but the smearing of strong discontinuities. Unsteady lift was well predicted for a two-dimensional flat-plate airfoil and for an oscillating rectangular wing.

CAP-TSD results for the thin F-5 wing were in good overall agreement with experimental steady and unsteady pressures and with a steady flow Euler code. One case with an embedded swept shock near the lower surface leading edge indicated that embedded transonic flows can be treated.

Results for the RAE tailplane model were in good agreement with the measured data for both the steady and unsteady cases and with a steady calculation with an Euler code. Good agreement was also found for a steady flow case at a Mach number of 1.71 and 5-deg mean angle of attack.

The present study has demonstrated the applicability of the CAP-TSD code to flows with supersonic freestream with favorable comparisons of selected cases. Improvements to the shock-capturing characteristics and the supersonic outer boundary conditions for supersonic flow are desirable.

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